Basics

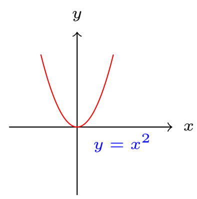
• Functions: are maps in which every x value has only one image $f(x) = y$

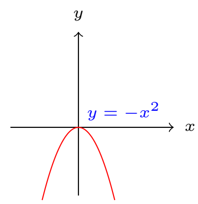
•y-intercept: Where f crosses y-axis $\to$ Let $x = 0$, then find $y = f(0)$

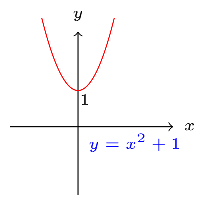
•x-intercept (zero or root): Where f crosses x-axis $\to$ Let $y = 0$, then find x

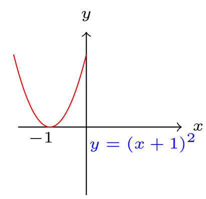
• Shifting and reflections: Given a function $y = f(x)$ and a constant $c > 0$, then

1) $y = f(x) + c$: Shift the graph of $f(x)$ c units upward.
2) $y = f(x) - c$: Shift the graph of $f(x)$ c units downward.
3) $y = f(x + c)$: Shift the graph of $f(x)$ c units leftward.
4) $y = f(x - c)$: Shift the graph of $f(x)$ c units rightward.
5) $y = -f(x)$: Reflect the graph of $f(x)$ about x-axis.
6) $y = f(-x)$: Reflect the graph of $f(x)$ about y-axis









• Linear functions (Lines):

• General Form: $y = f(x) = mx + b$, where $m = \frac{\Delta y}{\Delta x} = y'$ is the slope of the line.

• $(y - y_0) = m(x - x_0)$: Gives the equation of the line with slope m and passes through (x_0, y_0)

•: Horizontal line: $y = c \to$ Slope $= 0$

•: Vertical line: $x = c \to$ Slope undefined

•: If L_1 and L_2 are two lines with slopes m_1 and m_2 respectively, then

1) L_1 and L_2 are parallel if $m_1 = m_2$
2) L_1 and L_2 are perpendicular (normal) if $m_1 = -\frac{1}{m_2}$

• Solving Equations and inequalities with absolute value:

- $|x| = a \to x = \pm a$
- $|x| \leq a \to -a \leq x \leq a$
- $|x| \geq a \to x \leq -a$ or $x \geq a$

• Special Factorizations:

- $x^2 - a^2 = (x - a)(x + a)$
- $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$
- $x^3 + a^3 = (x + a)(x^2 - ax + a^2)$

- Quadratic functions (Parabolas):
- General Form: $y = f(x) = ax^2 + bx + c$; $a \neq 0$
- Vertex: is the point $(\frac{-b}{2a}, f(\frac{-b}{2a}))$
- Discriminant$= b^2 - 4ac$

  1) If discriminant > 0, then $f(x)$ has two real roots.

  2) If discriminant $= 0$, then $f(x)$ has one real root.

  3) If discriminant < 0, then $f(x)$ has no real roots.

- Quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

  If $a > 0$ then the parabola is open upward (concave up)

  If $a < 0$ then the parabola is open downward (concave down)

- Square Completion: Given $x^2 + bx + c$, (notice that $a = 1$), add $\pm(\frac{b}{2})^2$

  $\rightarrow x^2 + bx + c = x^2 + bx + (\frac{b}{2})^2 - (\frac{b}{2})^2 + c = (x - |\frac{b}{2}|)^2 - (\frac{b}{2})^2 + c$

  Ex: $x^2 - 6x + 11 = x^2 - 6x + 9 - 9 + 11 = (x - 3)^2 + 2$

- Special Quadratic Curves in y: $x = y^2$ and $x = -y^2$

  $x = y^2$: a parabola open to the right with vertex $(0, 0)$

  $x = -y^2$: a parabola open to the left with vertex $(0, 0)$

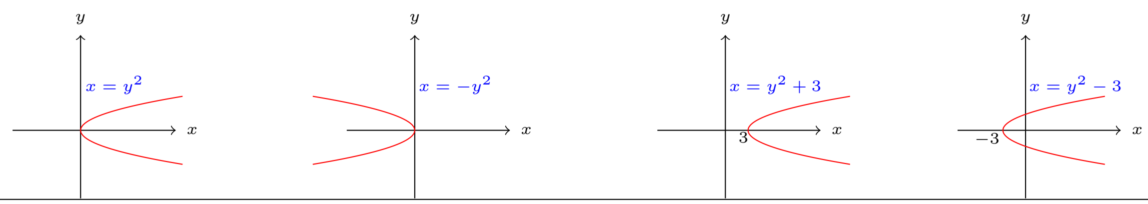
  Examples of shifts on $x = y^2$:

  1) $x = y^2 + 3$: Shift the graph of $x = y^2$ three units to the right

  2) $x = y^2 - 3$: Shift the graph of $x = y^2$ three units to the left

  3) $x = (y + 3)^2$: Shift the graph of $x = y^2$ three units downward

  4) $x = (y - 3)^2$: Shift the graph of $x = y^2$ three units upward



- Circles:

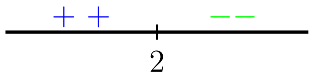
  $(x - a)^2 + (y - b)^2 = r^2$: a circle with center (a, b) and radius r

- Unit circle: $x^2 + y^2 = 1$: center$= (0, 0)$ and radius $= 1$

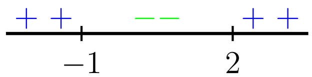
- Determine the sign of $y = f(x)$: Sometimes we need to know when y is positive (above x-axis) and when y is negative (below x-axis)

1) Polynomials: Find the zeros, if any, then substitute values

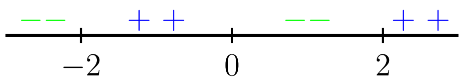
Ex: $f(x) = 4 - 2x \to 4 - 2x = 0 \to x = 2$ (take $f(0) = 4 > 0$ but $f(3) = -2 < 0$)



Ex: $f(x) = x^2 - x - 2 \to x^2 - x - 2 = 0 \to x = -1, 2$
$(f(-2) = 4 > 0$, $f(0) = -2 < 0$, $f(3) = 4 > 0)$



Ex: $f(x) = x^3 - 4x \to x^3 - 4x = 0 \to x = -2, 0, 2$



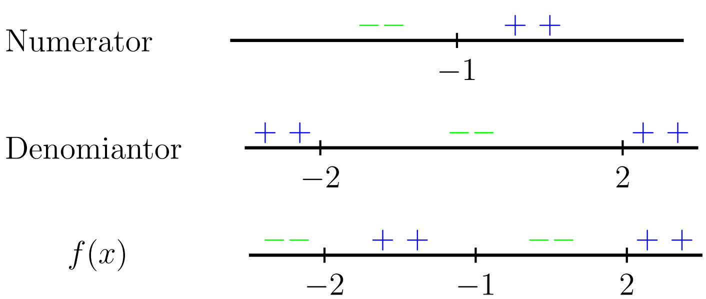
Ex: $f(x) = x^2 + 3$ has no zeros, so substitute any value $f(1) = 4 > 0$

2) Rational functions $= \frac{\text{polynomial}}{\text{polynomial}}$: Determine sign of numerator, then denominator, then divide

Ex: $f(x) = \frac{x^3+1}{x^2-4}$

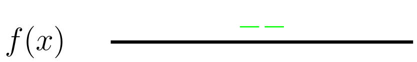
Numerator: $x^3 + 1 = 0 \to x = -1$

Denominator: $x^2 - 4 = 0 \to x = -2, 2$

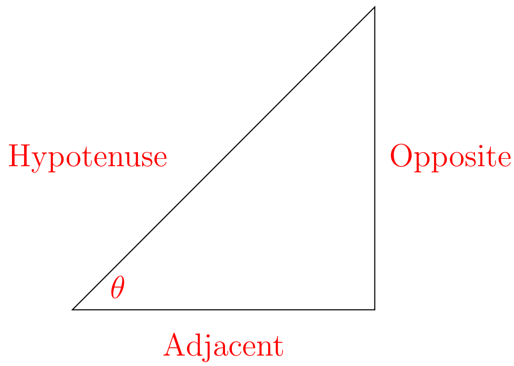


Ex: $f(x) = \frac{-2}{x^2+1}$

'The numerator is always negative and the denominator is always positive, so f is always negative.



- Trigonometric functions



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}} = \frac{1}{\cos\theta} \qquad \csc\theta = \frac{\text{hypotenuse}}{\text{opposite}} = \frac{1}{\sin\theta}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin\theta}{\cos\theta} = \frac{1}{\cot\theta} \qquad \cot\theta = \frac{\text{adjacent}}{\text{opposite}} = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta}$$

θ	$\sin\theta$	$\cos\theta$
0	0	1
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1	0
π	0	-1
$\frac{3\pi}{2}$	-1	0
2π	0	1

- Unit Circle and trigonometric functions:

Recall: Unit Circle: $x^2 + y^2 = 1$ and $\cos^2\theta + \sin^2\theta = 1$

$\to$ For any point on this circle: $(x, y) = (\cos\theta, \sin\theta)$,where θ : is the angle (counterclockwise) between the positive x-axis and the line segment form origin to point (x, y)

Ex: $(\frac{\sqrt{3}}{2}, \frac{1}{2}) = (\cos(\frac{\pi}{6}), \sin(\frac{\pi}{6}))$, $(0, 1) = (\cos(\frac{\pi}{2}), \sin(\frac{\pi}{2}))$, $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}) = (\cos(\frac{3\pi}{4}), \sin(\frac{3\pi}{4}))$